Macroscopic quantum tunneling in high-T_c superconductors: the role of collective charge oscillations

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We present a theoretical study of an escape rate for switching from the superconducting state to a resistive one in series arrays of strongly interacting Josephson junctions. At low temperatures such a switching is determined by macroscopic quantum tunneling (MQT) of a spatially extended Josephson phase. An increase of the crossover temperature from the thermal to quantum regimes, and a giant enhancement of MQT escape rate is obtained. Such an effect is explained by excitation of collective charge oscillations forming a *charge instanton*. Using a model of screened Coulomb interaction we found that our analysis is in a good accord with recently published experimental results on an enhancement of the MQT in single crystals of high-T_c superconductors.

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Great attention has been devoted to an experimental and theoretical study of diverse *macroscopic quantum phenomena*, e.g. macroscopic quantum tunneling (MQT) and energy level quantization just to name a few, in Josephson coupled systems [1, 2, 3, 4, 5, 6, 7, 8, 9]. A lumped Josephson junction is characterized by a single Josephson phase, and the MQT of a Josephson phase has been found long time ago in Nb Josephson junction [1, 2]. At low temperatures the MQT determines the escape rate of the switching from the superconducting state to a resistive one. The MQT is considered as a basic effect in the modern field of quantum information processing [10]

Next step in a study of macroscopic quantum phenomena is to obtain the MQT in *spatially extended* Josephson systems, i.e. dc SQUIDs [3], quasi-one-dimensional long Josephson junctions [4, 5, 6, 7], Josephson parallel and series arrays [8, 9]. These systems that can be characterized by coordinate and time dependent Josephson phase $\varphi(x,t)$, present a particular case of interacting many particle systems. The presence of inductive interaction in Josephson parallel arrays and long Josephson junctions allows one to form diverse macroscopic objects (solitons), e.g. Josephson vortices (magnetic fluxons) and vortex-antivortex pairs. Thus, the MQT in spatially extended Josephson systems can be often reduced to the quantum fluctuation induced escape of solitons from a pinning potential [4, 5, 6, 7]. Indeed, both the MQT of a Josephson vortex [6] and the quantum dissociation of a vortex-antivortex pair [7] have been observed.

Other example of spatially extended Josephson systems is a dc biased series array of Josephson junctions. This case presents a special interest because artificial series arrays of $Al/Al_20_3/Al$ junctions have been prepared [11], and layered high-T_c superconductors can be modelled as a stack of intrinsic Josephson junctions [12]. Moreover, modern fabrication technique allows to prepare single crystals of layered high-T_c superconductors with an extremely homogeneous distribution of critical currents of intrinsic Josephson junctions, and a low level of dissipation [8, 9]. Such systems are suitable for an experimental study of macroscopic quantum phenomena, and indeed, the MQT of a Josephson phase has been observed in layered $BiSrCaCuO$ superconductors [8, 9].

A rather unexpected result found in the Refs. [8, 9] is that the crossover temperature T^* from the thermal fluctuation regime to the MQT regime is much larger in respect to a single Josephson junction having the same parameters. Moreover, in Ref. [9] it has been observed that the MQT escape rate Γ_{MQT} for a stack of intrinsic Josephson junctions is *four order of magnitude* larger than Γ_{MQT} for a single Josephson junction. In these experiments it was also found that the escape rate Γ_T in the thermal fluctuation regime did not differ from the escape rate of a single Josephson junction.

Since in the model of independent Josephson junctions the crossover temperature does not depend on number N of junctions, and the escape rate Γ is just proportional to N, an enhancement of the MQT observed in layered high-T_c superconductors stems from an interaction between intrinsic Josephson junctions. In order to explain such a giant increase of the MQT escape rate the idea of a long-range coupling between intrinsic Josephson junctions has been proposed in Ref. [9]. Although this mechanism can definitely take place, we notice that a natural source of interaction in series junction arrays is the screened Coulomb charge interaction [13]. This interaction is especially strong in layered high-T_c superconductors, where the Debye screening length is of the order of superconducting layer thickness.

In this Letter we will show that the excitation of collective charge oscillations in a stack of Josephson junctions leads to an increase of the crossover temperature T^* and a giant enhancement of the MQT escape rate. In order to quantitatively analyze the escape rate in layered superconductors a series array of interacting Josephson junctions biased by dc current I (see schematic in Fig. 1) is considered. The system is characterized by the set of

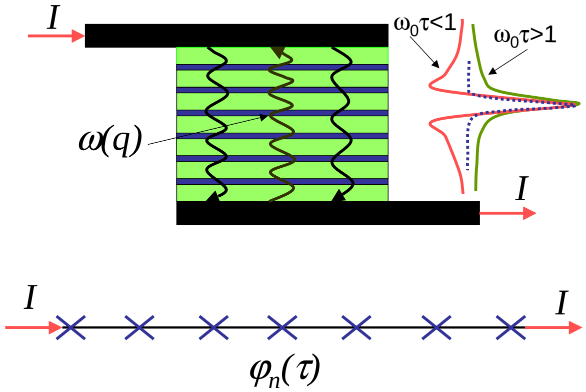

FIG. 1: Schematic of a dc biased layered high-$T-c$ superconductor and a series array of Josephson junctions. Collective charge oscillations (wave lines), a strongly localized instanton (dashed line) and a charge instanton with long tails (solid lines) are shown.

Josephson phases $\varphi_n(\tau)$, where number n changes from 0 to N. In order to obtain the escape rate Γ we use "instanton technique" [14, 15, 16], and therefore, τ is the imaginary time that varies from 0 to $\hbar/k_bT$ (T is the temperature). In the imaginary time representation the Lagrangian of a series array with a charge interaction is written as

$$L = \sum_n \sum_m \frac{1}{2}[\dot{\varphi}_n(\tau) C_{nm} \dot{\varphi}_m(\tau)] + \sum_n U_n ,$$

$$U_n(\varphi) = \cos\varphi_n(\tau) + i\varphi_n(\tau) , \tag{1}$$

where $i = I/I_c$ is the normalized external dc current, and I_c is the nominal value of the critical current of a single junction. The Lagrangian is expressed in units of E_J, where E_J is the Josephson energy of a single junction. The capacitance matrix $C_{nm} = C_{|n-m|}$ is determined by a charge interaction between junctions, and C_{nm} becomes the δ_{nm}-function for the specific case of decoupled junctions, i.e. as the charge interaction between junctions is small $C << C_0$. The C_0 is the capacitance of a single Josephson junction. In the experiments on the MQT in Josephson coupled systems $E_J >> E_{C_0} = e^2/C_0$, and the MQT occurs as the dc current I is close to I_c, and therefore $(i-1) \ll 1$. In this case the potential $U_n(\varphi)$ is written as

$$U_n(\varphi) = (1-i)\varphi_n(\tau) - \frac{\varphi_n^3(\tau)}{6} . \tag{2}$$

The escape rate is determined by the particular solution $\varphi_n(\tau)$ providing the extremum of action

$$S = -\frac{1}{\hbar}\int_0^{\hbar/k_BT} L(\tau)d\tau . \tag{3}$$

At high temperatures such a solution is determined by extremum points of the potential U_n, and it is written as

$$\varphi_n = 2\sqrt{2(1-i)}\delta_{nl} - \sqrt{2(1-i)} .$$

Here, l is a junction number where the fluctuation occurs. Since this solution does not depend on the time τ, we can immediately conclude that the excitation of charge oscillations does not change a slope of the dependence of the escape rate $\ln[\Gamma_T(1-i)]$ on the bias current $(1-i)$ in the thermal fluctuation regime. However, the crossover temperature from the thermal fluctuation regime to the MQT can be strongly enhanced by excitation of charge oscillations in a stack of interacting Josephson junctions. Indeed, using the method elaborated in [14, 15, 16] we obtain that at high temperatures the optimal fluctuation $\varphi_n(\tau)$ around an extremum point has a form:

$$\varphi_n(\tau) = e^{\frac{2\pi i k_B T\tau}{\hbar}}\phi_n , \tag{4}$$

where the eigenfunctions ϕ_n are the solution of the non-local and inhomogeneous equation:

$$\sum_m \frac{4\pi^2k_B^2T^2}{\hbar^2}C_{nm}\phi_m - 2\omega_0^2\delta_{lm}\phi_m = (\lambda-\omega_0^2)\phi_n . \tag{5}$$

Here, λ are the eigenvalues of the Eq. (5), $\omega_0 = \omega_p[2(1-i)]^{1/4}$ is the dc bias dependent frequency of oscillations on the bottom of potential well, $U_n(\varphi)$, and ω_p is the plasma frequency of a single Josephson junction. The crossover temperature T^* is determined by the condition that there is the eigenvalue $\lambda = 0$ [15, 16]. Using the Fourier transform of Eq. (5) the crossover temperature is obtained as a solution of the following equation:

$$1 = \int_0^{2\pi}\frac{dq}{2\pi}\frac{2\omega_0^2}{\omega_0^2+\xi^2[1+2C(q)]} , \; \xi = \frac{2\pi k_BT^*}{\hbar} . \tag{6}$$

Here, $C(q)$ is the Fourier cos-transform of the function C_{nm}, i.e. $C(q) = \frac{1}{C_0}\sum_{n=1}^{N} C_n\cos(qn)$. We can analyze Eq. (6) in different limits. In the case of decoupled junctions, i.e as $C(q) << 1$ for all q, we obtain that $T^*_{ind} = \hbar\omega_0/(2\pi k_B)$, and the crossover temperature of a series array T^*_{ind} coincides with the one for a single Josephson junction. However, for strongly coupled Josephson junctions $C(q)$ becomes negative for $q \simeq \pi$, and can be of the order of one. In this case, the crossover temperature has to increase.

This effect can be easily understood if we notice that the spectrum of charge oscillations in a stack of Josephson junctions forms a band

$$\omega^2(q) = \frac{\omega_0^2}{1+2C(q)} . \tag{7}$$

The crossover temperature is determined by the *maximum* value of the $\omega(q)$. Therefore, the excitation of high-frequency charge oscillations leads to an increase of the crossover temperature. On other hand a well-known effect of an enhancement of the MQT escape rate under microwave radiation [9] is determined in Josephson junction series arrays with a strong charge interaction by the

minimum value of the ω_q. Therefore, the value of the plasma frequency found from this effect is not directly related to the crossover temperature.

Now we turn to the MQT regime, where the extremum point of the action S is the "tau-dependent" instanton (bounce) solution. In the absence of a charge interaction the instanton solution is strongly localized on a particular junction (see schematic in Fig. 1, dashed line), i.e [16].

$$\varphi_n(\tau) = f_0(\tau) = \delta_{nl} \frac{3\sqrt{2(1-i)}}{\cosh^2(\omega_0\tau/2)} \quad . \tag{8}$$

In a generic case of interacting junctions , an instanton dressed by collective charge oscillations has small tau-dependent spatial tails (see Fig. 1, solid line). Such a spatial-temporal instanton solution that can be called a *charge instanton*, is found by perturbation analysis ($C(q) \leq 1$) as

$$\varphi_n(t) = \int_0^{2\pi} \frac{dq}{2\pi} \int\int \frac{d\omega d\tau_1}{2\pi} \frac{2C(q)e^{i\omega(\tau-\tau_1)}}{\omega_0^2 + \omega^2[1+2C(q)]} \ddot{f}_0(\tau_1) \quad . \tag{9}$$

Substituting (9) in the expression for action S (3) we obtain the escape rate Γ_{MQT} (in physical units) as

$$\Gamma_{MQT} = \Gamma_0 \exp\left[-\frac{72E_J}{15\hbar\omega_p} 2^{1/4}(1-i)^{5/4}(1-\chi)\right] , \tag{10}$$

where

$$\chi = 60\pi \int_0^{2\pi} \frac{dq}{2\pi} \int dx \frac{C^2(q)x^6}{\{x^2[1+2C(q)]+1\}\sinh^2(\pi x)} \tag{11}$$

The parameter χ having a positive value, characterizes an enhancement of MQT due to the presence of the charge interaction between Josephson junctions, as $C(q)$ is different from zero. The integral over x can be calculated by taking into account that the typical values of x are numerically small, i.e $x \leq (1/\pi)$. Therefore, with a good accuracy we obtain

$$\chi = \frac{20}{7} \int_0^{2\pi} C^2(q) \frac{dq}{2\pi} \tag{12}$$

Thus, remarkably the crossover from the thermal fluctuation regime to the MQT regime in Josephson series arrays with a charge interaction is realized as a transition from 0- to 2-dimensional instanton solution.

Presented above a generic analysis can be used for a particular case of the MQT in a single crystal of high-T_c superconductors contained intrinsic Josephson junctions. A natural source of a charge interaction in layered superconductors is the Coulomb screened charge interaction, and we use a simple model introduced by M. Tachiki et al. in Ref. [13]. In this model the function $C(q)$ has a form:

$$C(q) = -\frac{\alpha}{2} \frac{1-\cos(q)}{1+\alpha(1-\cos(q))} , \tag{13}$$

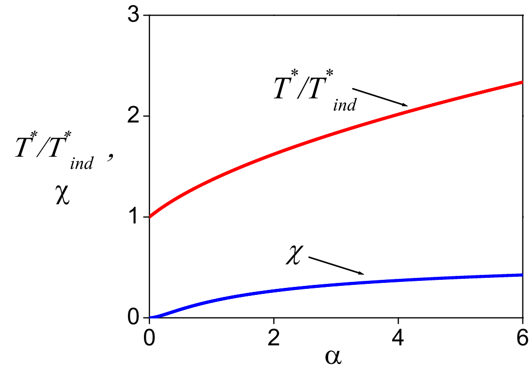

FIG. 2: The dependencies of the crossover temperature $T^*(\alpha)$ and the parameter $\chi(\alpha)$ on the strength of Coulomb interaction α.

where the parameter α characterizes a strength of the charge interaction. This parameter is determined by the ratio between the Debye screening length and a superconducting layer thickness, and α can be larger than one in layered high-T_c superconductors [13]. Substituting (13) in (6) and (12), and calculating the integrals over q, we obtain the dependence of the crossover temperature T^* and the parameter χ on the strength of Coulomb interaction α:

$$T^*(\alpha) = T^*_{ind}\sqrt{\frac{1+\alpha+\sqrt{4+\alpha^2+8\alpha}}{3}} \tag{14}$$

and

$$\chi = \frac{5}{7}\left[1 - \frac{1+3\alpha}{(1+2\alpha)^{3/2}}\right] \quad . \tag{15}$$

These dependencies are shown in Fig. 2.

Moreover, in Fig. 3 we present the dependence of the MQT escape rate on the external dc bias for various values of α. One can see a giant enhancement of the MQT escape rate as the parameter α becomes larger than one. This enhancement results from a decrease of the slope of the bias current dependence escape rate $\ln[\Gamma_T(1-i)]$ on $(1-i)$. Comparing our theoretical predictions with the experimental curves published in Ref. [9] (see Fig. 5 in Ref. [9]) we find a good agreement as $\alpha \simeq 4$. By making use of the same value of α we obtain that the crossover temperature increases two times in respect to a single Josephson junction with the same parameters.

Notice here that for a finite system as a number of intrinsic Josephson junctions N is not very large, there is a discrete number of collective oscillation modes that can be excited in a system. In this case the integral over q in Eq. (12) has to be changed to the sum over discrete modes, i. e. $\int dq/(2\pi) \to (1/N)\sum_n$. Therefore, the slope of the bias current dependence escape rate

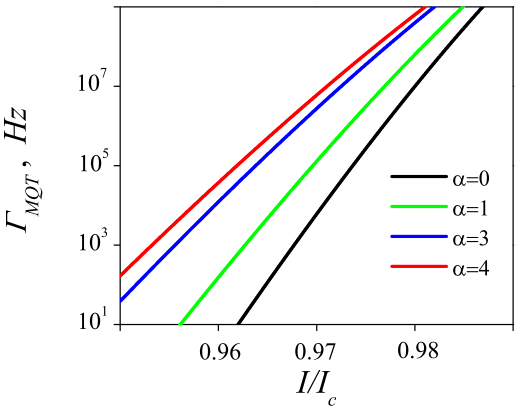

FIG. 3: The dependence of the MQT escape rate Γ_{MQT} on the dc bias current I for various values of $\alpha = 0,\ 1,\ 3,\ 4$. Legend corresponds to the curves arranged from right to left. In order to fit our theoretical predictions to the experimental results of Ref. [9] the value of $E_J/(\hbar\omega_p) \simeq 264$ and preexponent in the expression (10) $\Gamma_0 = 1\ GHz$ were used.

$\ln[\Gamma_T(1-i)]$ increases for a series array with less number of Josephson junctions. It gives a natural explanation of a moderate enhancement of the MQT escape rate as a number of intrinsic Josephson junction N was decreased [9].

In conclusion we have shown that the MQT escape rate can be drastically increased in strongly coupled Josephson junctions series arrays. The MQT in such arrays is described through the quantum fluctuation induced excitation of a charge instanton characterized by long spatial tails. It is in marked contrast to the well-known cases of a single Josephson junction or weakly coupled Josephson junctions series arrays where a strongly localized instanton solution forms. Appearance of the charge instanton is explained by excitation of collective charge oscillations, and this mechanism has to be especially effective in layered high-T_c superconductors where the screening of Coulomb interaction is rather weak. The excitation of high-frequency charge oscillations also leads to a significant increase of the crossover temperature from the thermal fluctuation to the quantum regime. Such an enhancement of the MQT in Josephson series arrays with a strong charge interaction can be very promising for a field of quantum information processing [10].

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